Dr. Eakin's MA 110 Exams From Fall 2007:

CAUTION: MA110 is not static: the course content, text, homework, and syllabus change from year to year. While the exam format will be the same, there should be no expectation that the Fall 2008 exams reflect those of previous years in content or problem selection.

Exam 1:
Math 110-001, 002, 005
September 21, 2007

Instructions: This examination consists of nine (9) questions on six (6) pages. Each question counts 10 points.
Please make sure you have a complete exam. Enter your name, section, and TA information at the top of this page and your name or initials on each of the other pages.

Please note that, you are expected to justify your answers by showing your work. **Your work is what will be graded.**
Unsupported answers are not worth anything.
"**Answers" simply taken from calculators or estimated from graphs will receive no credit.**
(It is , of course, quite reasonable to use your calculator to cross-check your answers.)

Partial credit may be assigned.
The papers must be turned in at the end of class time. The scores on the in-class portion will be curved

Problem	*Score*
1	________
2	________
3	________
4	________
5	________
6	________
7	________
8	________
9	________
Homework	________
Total	________

(1) (10 points) Write $\frac{17}{23}-\frac{31}{19}$ in the form $\frac{A}{B}$ where A and B are **<u>integers.</u>** You do <u>not</u> have to express this fraction in lowest terms! (SHOW YOUR WORK!)

(2) (10 points) If $A=\frac{1}{5}$, $B=\frac{1}{4}$, and $\frac{X+1}{X-2}=\frac{A}{B}$ then X = ______________________

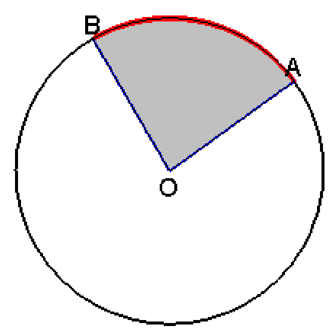
Note that X must be expressed as a **rational number** (a ratio of two integers, <u>not</u> as a decimal. (SHOW YOUR WORK!)

(3) Decimal answers are fine for this problem. SHOW YOUR WORK

The circle in the diagram is centered at O and has a radius of 10 feet. If the length of the arc AB is 15 feet then:

a. (6 points) The angle AOB measures ______________ radians or ________________ degrees. (SHOW YOUR WORK!

b. (4 points) The area of the colored sector is ______________________ square feet. (SHOW YOUR WORK!)



(4)

(a) (5 points) If $A = 3-5\,i$ is a complex number then $\frac{1}{A}$ = ______________ + ______________ i (SHOW YOUR WORK!)

(b) (5 points) If X is a complex number such that $(3-5\,i)\,X = 11-7\,i$ then X = ______________________ (SHOW YOUR WORK!)

(5) (10 points) The coefficient of x^4 when $(x^3-5\,x^2+7\,x-1)\,(2\,x^2+x-1)$ is multiplied out and simplified is _______________ (SHOW YOUR WORK!)

(6)
(a) (5 points) If $(2A-3B)^5$ is expanded and simplified then the coefficient of A^2B^3 will be ________________________ (SHOW YOUR WORK!)

(b) (5 points) If $(x-y)^{17}$ is expanded and simplified then the coefficeint of $x^{14}y^3$ will be ____________________ (SHOW YOUR WORK!)

(7) (10 points) { $2x+3y=11$, $4x+2y=6$ } is a system of two linear equaitons in the variables x and y.
Use Cramer's Rule to determine x. Note <u>you must use Cramer's Rule</u> to get credit for the problem. (SHOW YOUR WORK!)

(8) (10 points) The sum of two numbers is 7. Twice one of them plus three times the other is 9. . The numbers are ________________ and _____________________ (List them in either order. SHOW YOUR WORK!)

(9).
(a) (4 points) The determinant of the matrix $\begin{bmatrix} 7 & -3 \\ 4 & 5 \end{bmatrix}$ is ______________________
(SHOW YOUR WORK!)

(b) (6 points) If $f(x)=x^5-4x^3-5x+12$ and $g(x)=-x^3+x^2+1$ then the degree of $f(x)\,g(x)$ is ________________ and the degree of $f(x)+x^2\,g(x)$ is __________________
(SHOW YOUR WORK!

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Exam 2:
Math 110-001, 002, 005
October 19, 2007

Instructions: (omitted)

1. The line L has two coordinate systems. In one system the coordinates of a point P are represented by the variable x . In that system Z = Z(3) would denote that the x-coordinate of the point Z is 3. In the other system the coordinates are represented by the variable x ' . In it , Z =

Z '(7) would denote that the x ' coordinate of the point Z is 7. As indicated in the table below, the origin in x - coordinates is the point O and the unit point in x- coordinates is the point P. Also, as indicated in the table, the origin in x ' coordinates is A and the unit in x ' coordinates is B.

a. (4 points) <u>Note that there are two answers to part a of this problem.</u>
Use the information in the table to determine a formula of the form $x' = a\,x + b$ for translating from x- coordinates to x ' coordinates.
Also, give a general formula of the same type for transforming from x ' coordinates to x-coordinates .

$$\begin{bmatrix} point & x\text{- } coordinate & x' \; coordinate \\ \mathrm{O} & 0 & \_\_\_\_\_\_ \\ P & 1 & \_\_\_\_\_\_ \\ A & 3 & 0 \\ B & \frac{5}{2} & 1 \\ R & \_\_\_\_\_\_ & 5 \end{bmatrix}$$

b. (6 points) Fill in the missing entries in the table. <u>There are three numbers to enter</u>.

2. The point P(x,y) in the plane can be thought of as the complex number $x + i\,y$. If C is any complex number then there is a transformation of the plane defined by $\mathrm{T}(P) = C * (x + i\,y)$. Suppose T is such a transformation and T takes the point A = A(2,3) to T(A) = T(A)(14,-5). That is, the point with coordinates (2,3) is taken to the point with coordinates (14,-5).

a. (5 points) What is the complex number C?

b. (5 points) If B = B(-7,1) and T(B) = T(B)(z,w) then what are z and w? That is, what are the coordinates of T(B)?
ANSWER T(B) = (___________, _____________)

3. For this problem 8 km = 5 miles
Town A is in a French-speaking country and town B is in an English speaking country. They are 100 miles apart and connected by a straight road. When giving directions to a point on the road between them the people in town A will say that it "x km from here "(i.e. from town A) while the people in town B will say that it is "x ' miles from here" (i.e. from town B). Thus every point on the road has an x- coordinate given it by town A and an x'coordinate given it by town B.

a. (5 points) If the border is 30 miles from town A the people in town B will say that it is ___________ " miles from here".

b. (5 points) Give a general formula for calculating the x ' (miles from B) coordinate in terms of the x (km from A) coordinate.
ANSWER x ' = ______________ * x + ___________________

4.
(a) (3 points) Recall that if a and b are integers then a|b means that a divides b (i.e. a is a factor of b).
The integer d is said to be the **greatest common divisor** of the integers m and n if:
i. $0 \le d$,
ii: d | m and d | n, and
iii. if q|m and q|n then ______________

(b) (2 points) Suppose we have a polynomial $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$ and we know that it is monic of degree 3.
Then we know a_3 = _________ and a_4 = ___________

(c) (2 points) Recall that if a and b are integers then a|b means that a divides b (i.e. a is a factor of b)..
The integer L is the **least common multiple** of the integers m and n if
i. $0 \le L$,
ii. m| L and n| L, and
iii if m | q and n | q then ___________________

(d) (3 points) Suppose f(x), g(x) and d(x) are polynomials with f(x) of degree 7, g(x) of degree 5, and d(x) is of degree 3 and we use long division to write $f(x)\,g(x) = d(x)\,q(x) + r(x)$.

The possible degrees for the remainder, r(x), are _____________________ and the degree of q(x), the quotient is __________.

5. a. (6 points) If $2567 = a_0 + a_1 5 + a_2\ 5^2 + a_3\ 5^3 + a_4\ 5^4 + a_5\ 5^5 + a_6\ 5^6 + a_7\ 5^7$ and each of $a_0, a_1, \ldots, a_7$ is one of the integers { 0,1,2,3,4} then
a_0 = __________, and a_1 = ___________

b. (4 points) If today is Friday then the day of the week 596 days from now will be _____________

6.
a. (6 points) Complete the Arybhata algorithm table and use it to answer the questions below

$$\begin{bmatrix} & 1 & 0 & 268 \\ -1 & 0 & 1 & 244 \\ -10 & 1 & -1 & 24 \\ & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

b. (1 point) The GCD of 268 and 244 is d = _______________________

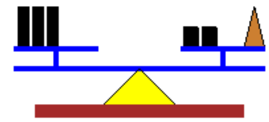
c. (2 points) d = ____________________ *268 + ____________________* 244

d. (1 point) The least common multiple of 268 and 244 is ______________________

7. Suppose you have a large collection of **33** gram weights and another large collection of **39** gram weights.
You have a pan balance and, as indicated in the diagram you want to use it to check that the weight of an object is exactly **15** grams.

a. (8 points) Use the table below to explain how to do this. Note the object (the cone in the diagram) is in the right pan so you must <u>explain</u> <u>how many of each kind of weight</u> to add to <u>each pan</u> if the object weghs exactly 15 grams and <u>**why it will balance**</u>.

$$\begin{bmatrix} & 1 & 0 & 39 \\ -1 & 0 & 1 & 33 \\ -5 & 1 & -1 & 6 \\ -2 & -5 & 6 & 3 \\ & 11 & -13 & 0 \end{bmatrix}$$



b. (2 points) Explain why it is not possible to check the weight of a 2 gram object with this set of weights.

8.

a. (4 points)

Complete the following Arybhata algorithm table and use it to answer the questions below. <u>You do not need to show work on this problem.</u>

$$\begin{bmatrix} , & , & 1\,, & 0\,, & , & x^5-x^4-x^2+x \\ -\frac{x}{2}\,, & , & 0\,, & 1\,, & , & 2x^4-2x^3-x \\ 4x^2+4x+8\,, & , & 1\,, & -\frac{x}{2}\,, & , & -\frac{x^2}{2}+x \\ -\frac{1}{7}+\frac{x}{14}\,, & , & 4x^2+4x+8\,, & \underline{\hspace{4cm}} & , & ,\ 7x \\ , & , & \frac{2x^3}{7}-\frac{2x^2}{7}-\frac{1}{7}\,, & -\frac{x^4}{7}+\frac{x^3}{7}+\frac{x}{7}-\frac{1}{7}\,, & , & 0 \end{bmatrix}$$

b. (2 point)   If   f(x) = $x^5-x^4-x^2+x$ and g(x) = $2x^4-2x^3-x$ then

the MONIC GCD of f(x) and g(x) is   ___________________________________

c. (2 points) Enter polynomials in the blanks

If d(x) is the MONIC gcd of $x^5-x^4-x^2+x$ and $2x^4-2x^3-x$ then

d(x) = _____________________*($x^5-x^4-x^2+x$) + ____________________*($2x^4-2x^3-x$)

d. (2 points) Enter a constant in the first blank below and a polynomial in the second. (This can be done in more than one way.)

The **MONIC** least common multiple of   f(x) = $x^5-x^4-x^2+x$ and g(x) = $2x^4-2x^3-x$ is

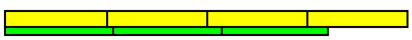
L(x) = __________ ($\frac{2x^3}{7} - 2x^2 - 1$)*(___________________________). [Do <u>not</u> multiply this out.]

9. A walkway is paved with a row of blue bricks that are each 32 inches long alongside a parallel row of yellow bricks that are each 22 inches long. At each end of the walkway the ends of the bricks are perfectly lined up.

a. (4 points) How many yellow bricks are in the walkway?

b. (3 points) How many blue bricks are in the walkway?

c. (3 points) How long (in inches) is the walkway?



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Exam 3:
Math 110-001, 002, 005
November 16, 2007

Instructions: (omitted)

1. (10 points) Suppose A = (3,7) and B = (5,11). SHOW YOUR WORK.

Give parametric equations, x(t), y(t) for the line passing through A and B
x(t) = _______________________
y(t) = _______________________

2. (10 points) If the line L is described by the parametric equations $x(t) = 2t + 1,\ y(t) = 4t - 3$ then the slope-intercept equation of L is y = ______ x + _________ . SHOW YOUR WORK.

3. (10 points) Give parametric equations for the **perpendicular bisector** of the line segment with end points A = (3,7) and B = (5,11) SHOW YOUR WORK.

x(t) = ________________________

y(t) = ________________________

4. (10 points) The graphs of a line L and a parabola P are given in the diagram below.
Circle the correct response. If there is insufficient information then circle "I"
You do not need to show work for this problem.

(a) If L is the graph of y = mx+b then:

i. The sign of m is: + - 0 I

ii. The sign of $-\frac{b}{2a}$ is: + - 0 I

iii. The sign of -b/m is: + - 0 I

(b) If P is the graph of $y = ax^2 + bx + c$ then

i. The sign of a is: + - 0 I

ii. The sign of b is: + - 0 I

iii. The sign of $b^2 - 4ac$ is: + - 0 I

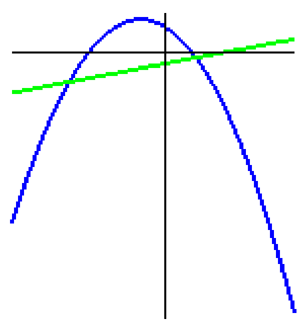
iv. The sign of c is: + - 0 I

v. The sign of b is: + - 0 I

vi. If r_1 and r_2 are the values of x such that $ax^2 + bx + c = 0$ then the sign of $\frac{r_1 + r_2}{2}$ is:

\+ - 0 I

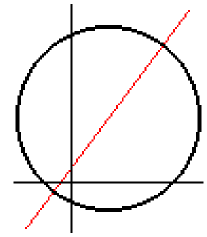
vii. The sign of the largest possible value of $ax^2 + bx + c$ is: + - 0 I



5. (10 points) A line with slope 5 is perpendicular to the line which passes through the points (1,-2) and (6,t). The value of t is __________ SHOW YOUR WORK.

6. The line L is given parametrically by the equations $x(t) = 3t + 4,\ y(t) = 4t + 7$
C is the circle of radius 10 with center at (4,7).

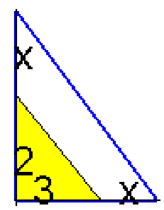
(a). (8 points) What are the values of the parameter t which correspond to the points on L at which L meets C? SHOW YOUR WORK.



(b). (2 points) What are the points of intersection of the line L and the circle C? Be sure to give both coordinates for each.
SHOW YOUR WORK

7. (10 points) The line L is decribed parametrically by the equations $x(t) = t + 3,\ y(t) = -2t + 3$. The line M is described parametrically by the equations $x(s) = -s + 2,\ y(s) = 3s + 4$. Find the point of intersection of L and M.
SHOW YOUR WORK.

8. The right triangle with sides of length 2 and 3 feet is expanded by adding a number x to each side. The area of the extended triangle is 45 equare feet.
(a) (6 points) Determine a quadratic equation $ax^2 + bx + c = 0$ for which the unknown x is a solution. SHOW YOUR WORK



(b)(4 points) What is the value of x? SHOW YOUR WORK.

9. (10 points) The parabola P is the graph of $y = x^2 + bx + c$. If (-1,9) and (1,3) are on P then b = _______ and c = __________

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Final Exam:
Math 110-001, 002, 005
December 14, 2007

Instructions: (omitted)

1.

(a) (5 points) If $A = 2 - 3\,i$ is a complex number then $\dfrac{1}{A}$ = ________________ + $\dfrac{3}{13}\;i$
SHOW YOUR WORK.

(b) (5 points) Enter a complex number $a + b\,i$ in the blank. Here a and b are rational numbers (which might or might not be whole numbers) (SHOW YOUR WORK!)
If X is a complex number such that $(2 - 3\,i)\,X = 29 - 11\,i$ then X = 7 + _____ i

(2) (10 points) { $3\,x - 5\,y = 8\,,\quad 2\,x - 4\,y = 4$ } is a system of two linear equaitons in the variables x and y.
Use Cramer's Rule to show that x = 6. Note <u>you must use Cramer's Rule</u> to get credit for the problem. (SHOW YOUR WORK!)

3. SHOW YOUR WORK!
a) (5 points) If $(3\,a - 2\,b)^7$ is expanded and simplified then the coefficient of $a^3\,b^4$ will be ________________________ (SHOW YOUR WORK!)

(b) (5 points) If $(x - y)^{19}$ is expanded and simplified then the coefficeint of $x^2\,y^{17}$ will be ____________________ (SHOW YOUR WORK!)

4.
a. (4 points) Complete the Arybhata algorithm table and use it to answer the questions below: SHOW YOUR WORK.

$$\begin{bmatrix} & 1 & 0 & 115 \\ -1 & 0 & 1 & 85 \\ \_\_\_ & 1 & -1 & 30 \\ -1 & \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & 3 & -4 & 5 \\ & -17 & 23 & 0 \end{bmatrix}$$

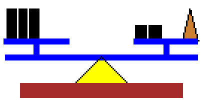
b. (2 points) The GCD of 115 and 85 is d = ______________________

c. (2 points) d = ___________________ *115 + ___________________* 85

d. (2 points) The least common multiple of 115 and 85 is _____________________

5. Suppose you have a large collection of 26 gram weights and another large collection of 34 gram weights.

(a) (7 points) You have a pan balance and, as indicated in the diagram you want to use it to check that the weight of an object is exactly
38 grams. Use the table below to explain how to do this.
Note the object is in the right pan so you <u>must describe how many of each kind of weight to add to each pan</u> so that it is guaranteed to balance .



$$\begin{bmatrix} & 1 & 0 & 34 \\ -1 & 0 & 1 & 26 \\ -3 & 1 & -1 & 8 \\ -4 & -3 & 4 & 2 \\ & 13 & -17 & 0 \end{bmatrix}$$

In the left pan place _____________ 26 gram weights and _________________ 34 gram weights.

In the right pan place the 38 gram object together with _____________ 26 gram weights and _________________ 34 gram weights

b. (3 points) Explain why it is not possible to check the weight of a 21 gram object with this set of weights.

6.
A line L has two coordinate systems. In one system the coordinates of a point P on the line are represented by the variable x and P = P(1) would denote that the x-coordinate of P is 1. In the other system the coordinates are represented by the variable x' and for thar system P = P'(-1) would denote that the x' coordinate of P is -1. The origin in x - coordinates is the point O and the unit point in x- coordinates is the point P. The origin in x ' coordinates is A and the unit in x' coordinates is B. As indicated in the table we have O = O'(2). Also, as indicated in the table, we have A = A(4) and B = B '(-2)

$$\begin{bmatrix} \textit{point} & \textit{x- coordinate} & \textit{x ' coordinate} \\ \mathrm{O} & 0 & 2 \\ P & 1 & -1 \\ A & 4 & \underline{\quad\quad} \\ B & \underline{\quad\quad} & -2 \end{bmatrix}$$

i. (3 points) Find a and b such that x' = a x+b is a formula for determining the x' coordinate of a point from the x coordinate of that point: SHOW YOU RWORK

a = ______________

b = ______________

ii. (4 points) Fill in the missing entries in the table. SHOW YOUR WORK

7. Suppose A and B are angles such that
sin(A) = .975 cos(A) = -.222
sin(B) = .663 cos(B) = .748 then:
a. (2 points) sin(A+B) = ___________________

b. (2 points) cos(A-B) = ____________________

c. (2 points) tan(A) = _______________________

d. (2 points) sin(P/2-A) = __________________

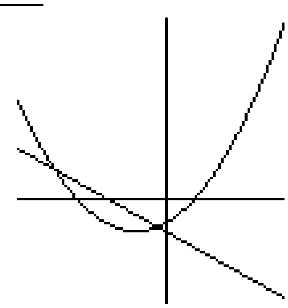
e. (2 points) csc(A-B) = _________________

8. YOU DO NOT NEED TO JUSTIFY ANSWERS IN THIS PROBLEM.
The graphs of a line L and a parabola P are given in the diagram below. Refer to the diagram and enter a "+", "-", oe "0" in each blank.
(a) If L is the graph of y = mx+n then
i. The sign of m is ________________
ii. The sign of n is ________________
iii. The sign of -n/m is _____________

(b) If P is the graph of $y = a x^2 + b x + c$ then
i. The sign of a is ___________________
ii. The sign of b is ___________________
iii. The sign of $b^2 - 4 a c$ is __________
iv. The sign of c is __________________
v. The sign of $-\frac{b}{2 a}$ is ________________

vi. If r_1 and r_2 are the values of x such that $a x^2 + b x + c = 0$ then the sign of $\frac{r_1 + r_2}{2}$ is __________

vii. The sign of the smallest possible value of $a x^2 + b x + c$ is __________



9. SHOW YOUE WORK
The line L is given parametrically by the equations: $x(t) = -3t + 1$, $y(t) = 2t - 5$.

The equation of the line M is $y = -\frac{x}{2} + 5$.

a. (5 points) The point of intersection of L and M is (__________,____________)

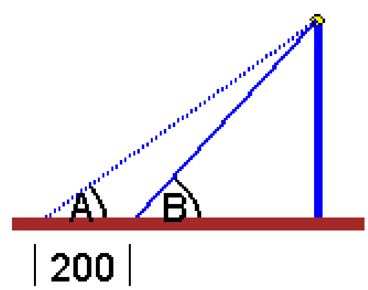
b. (5 points) M has the equation y = ___________x + ______________

10. Suppose A = (-4,9) and B = (2, 6). SHOW YOUR WORK

a. (5 points)Give parametric equations, x(t), y(t) for the line passing through A and B (There is more than one correct answer to this question.)
x(t) = _____________________
y(t) = _____________________

b. (5 points) The point on the line through A = (-4,9) and B = (2, 6) which is between A and B and whose distance from A is $\frac{1}{3}$ of the distance from A to B is (____________, _____________)

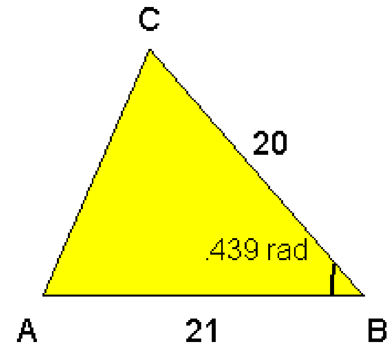
11. SHOW YOUR WORK.
At an unknown distance from its base, the top of a tower subtends an angle B which measures 60 degrees. From a distance along a line through the tower base and the first observation point and 200 feet further away the top of the tower subtends an angle A which measures 45 degrees .



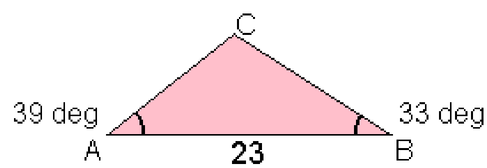
a. (5 points) The height of the tower is _________________________ feet.

b. (5 points) The distance from the vertex of the angle A to the base of the tower is _______ feet.

12. SHOW YOUR WORK!
a. (5 points) In the triangle ABC (as represented in the diagram) the angle B is .430 radians, the side BC has length 20 and the side AB has length 21. The length of the side AC (the one opposite the angle B) is ______________________.



b. (5 points) In the triangle ABC (as represented in the diagram) the angle A is 39 degrees, the side AB has length 23, and the angle B measures 33 degrees. The length of the side AC (the one opposite the angle B) is ______________________. SHOW YOUR WORK!



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