Final Exam Practice Problems
Spring 2008

WARNING: THIS IS SIMPLY A SMALL SET OF PROBLEMS OF TYPES THAT WOULD BE APPROPRIATE FOR THE FINAL EXAM. BE SURE TO REVIEW THE IN-CLASS TESTS, HOMEWORK, AND LECTURE NOTES

Instructions: This examination consists of xxx (xx) questions on xx (xx) pages.
Each problem is worth 10 points.
Please make sure you have a complete exam.
Enter your name, section, and TA information at upper right on this page and your name or initials on each of the other pages.

Please note that, except for question 1, you are expected to provide supporting calculations or otherwise explain how you arrived at your answers.

Expressions such as $13\ln\left(\frac{76}{12}\right)$, e^5, $5\ln(2)+\arctan\left(\frac{\pi}{4}\right)$, etc. that are part of final answers need not be evaluated on your calculator.

If you do prefer to work with floating point values then be sure the grader can determine what they represent.

Problem	*Score*
1	________
2	________
3	________
4	________
5	________
6	________
7	________
8	________
9	________
10	________
11	________
12	________
Homework	________
Total	________

1. (10 pts - 2 pts each) Respond as indicated to each of the following. **You do not need to show work for this problem.**

a. By definition $\sum_{i=1}^{\infty} a_i = \lim_{n \to \infty} \sum_{i=\_\_\_}^{\_\_\_}$ __________

b. According to the ratio test $\sum_{n=1}^{\infty} a_n$ is absolutely convergent if $\lim_{n \to \_\_\_}$ ________

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c. Suppose f(x) is continuous on $(0, \infty)$ and $f(i) = a_i$ for i = 1,2,... Suppose further that we are given that the integral test applies to $\sum_{i=1}^{\infty} a_i$ and that $\int_1^{\infty} f(x)\, dx < \infty$. If $R_n = \sum_{i=n+1}^{\infty} a_i$ is the remainder then we have the estimate $\int_{n+1}^{\infty} f(x)\, dx \le R_n \le \int_{\_\_\_}^{\infty} f(x)\, dx$

d. For which values of r does the series $\sum_{i=0}^{\infty} r^i$ converge absolutely? ______________

e. For which values of p does the series $\sum_{i=1}^{\infty} t^q$ converge absolutely? ______________

f. The alternating series $\sum_{n=1}^{\infty} a_n$ converges if $\lim_{i \to \infty} |a_i| = 0$ <u>and</u> ________________________

g. Suppose f(x) is a continuous function such that $\{ a_i \}$ is a sequence such that $a_i = f(i)$ for i = 1,2, ... and $\int_1^{\infty} f(x)\, dx < \infty$. The integral test can be used to determine whether $\sum_{i=1}^{\infty} a_i$ converges provided f(x) has two additional properties:
(i) $0 \le f(x)$ for all x and (b) ____

2. SHOW YOUR WORK ON BOTH PARTS OF THIS PROBLEM.

(a) (5 points) Determine whether the series $\sum_{n=2}^{\infty} \frac{n^2}{\ln(n)}$ is convergent or not.

(b) (5 points) For which values of z is the following series absolutely convergent

$\sum_{n=0}^{\infty} \frac{7\,(2-5\,z)^n}{n^2}$?

3. (5 points) Use the integral test to determine whether the series $\sum_{n=1}^{\infty} \frac{2\,n}{n^2+1}$ is convergent or divergent. <u>You must use the integral test and must show that it applies.</u>

4. (10 points) Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^3+n}$ is convergent or divergent by using the comparison test. <u>You must use the comparison test, and must verify that it applies, and must explain or demonstrate why the series being used for comparison is convergent or divergent.</u>

5. Calculate the exact values for each of the following

a. (5 points) $\sum_{i=3}^{\infty} \sin\left(\frac{\pi}{6}\right)^i$

b. (5 points) $\sum_{n=1}^{\infty} \frac{1}{n\,(n+3)} =$

6.

a. (5 points) What is the least n for which you can be certain by the alternating series test that $\sum_{i=1}^{n} \frac{(-1)^i}{i}$ is within

.05 of $\sum_{i=1}^{\infty} \frac{(-1)^i}{i}$?

b. (5 points) The partial sum, $S_8 = \sum_{j=1}^{8} a_j$ of the series $\sum_{j=1}^{\infty} (-1)^j j$ is

____

7. (10 points) A car travels counterclockwise around a circular track at a constant speed, circling the track six (6) times per hour. The track has a radius of 2 miles. If the track is assumed to be the graph of $x^2 + y^2 = 4$ and at time t=0 the car is at (0,2), then the position of the car at time t hours is given parametrically by
x(t) = ____
y(t) = ____

8. The curve C is described parametrically by $x(t) = t^2 + 3t + 1,\ y(t) = 3 + 4t + t^3$

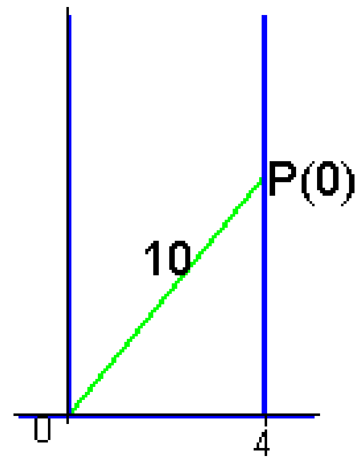
a. (3 points) Calculate a parametric form for the tangent line to C at the point corresponding to t=1.

b. (3 points) Calculate a parametric form for the normal line to C at the point corresponding to t=1.

b. (2 points) Calculate $\frac{dy}{dx}$ at the point where t = 1.

c. (2 points) Give a formula (in terms of the parameter t) for $\frac{dy^2}{dx^2}$. DO NOT SIMPLIFY YOUR ANSWER.

9. (10 points) A ladder 10 feet rests against the right wall of a narrow passageway with its base against the left wall. However the right wall is moving to the right at a constant rate of 3 feet per minute. At time t=0 the passageway is 4 feet wide. Give parametric equations describing P(t), the point of contact of the ladder with the right wall for times between t = 0 and t = 3 minutes. The diagram represents the time t = 0.



10 . Calculate the following. SHOW ALL WORK

a. (5 points) $\int \frac{1+\tan(2x)^2}{\sqrt{1+\tan(2x)}}\,dx =$ ______________________

b. (5 points) $\int_0^{\frac{\pi}{4}} \frac{\sin(x)}{4\cos(x)^2+1}\,dx =$ ______________________

11. Suppose $f(x)=\frac{2x+1}{3x+5}$ for x>5/3

a. (5 points) Calculate $f^{(-1)}(x)$

b. (5 points) For your function $f^{(-1)}(x)$ in part (a) show $f^{(-1)}(f(x)) = x$

12. Calculate the following:

a. $\int x\cos(x)\,dx$ = ______________________________

b. $\int x\ln(x)\,dx$ = ________________

c. $\int \sin(x)\, \mathbf{e}^{x}\, dx =$ ____________

d. $\int x^{3}\, \mathbf{e}^{x}\, dx =$ _________________-

13. (10 points) The table at right gives the values of a function f(x) and its first four derivatives at selected values of x.

x	-2	-1	0	1	2
$f(x)$	$\frac{7}{6}$	$\frac{3}{2}$	$\frac{5}{6}$	$\frac{1}{6}$	$-\frac{15}{2}$
$f'(x)$	$-\frac{5}{6}$	0	$-\frac{5}{6}$	$-\frac{4}{3}$	$-\frac{39}{2}$
$f^{(2)}$	$\frac{22}{3}$	-2	$\frac{2}{3}$	$-\frac{14}{3}$	-38
$f^{(3)}$	-22	0	2	-16	-54
$f^{(4)}$	32	12	-8	-28	-48

a. (3 points) Calculate the Taylor polynomial of order 3 for f(x) at x = 0. Do not simplify your answer.

b. (3 points) Calculate the Taylor polynomial of order3 for f(x) at x = 1 Do not simplify your answer.

c. (3 points) Calculate the Taylor polynomial of order 3 for f(x) at x = 1 Do not simplify your answer.

d . (2 points) Let $\mathrm{h}(x) = 3\, x\, \mathrm{f}(x)$. Calculate the Taylor polynomial of degree 3 for h(x) at x =1 Do not simplify your answer.

e . (2 points) Calculate the Taylor polynomial of degree 3 for f ' (x) (the derivative of f(x)) at x =0 Do not simplify your answer.

14.

a. (5 points) Calculate the trapezoid rule estimate for $\int_{-1}^{3} x^{3} + x\, dx$ with **two** sub-intervals.

Use the appropriate error formula to calculate the maximum error in this estimate.

b. (5 points) Calculate the Simpson's rule estimate for $\int_{-1}^{3} x^3 + x \, dx$ for 4 sub-intervals.

15 .

a. (5 pts) Use partial fractions to evaluate $\int \frac{1}{(x-5)(x-2)} \, dx$

b. (5 points) Calculate the partial fraction expansion of f(x) = $\frac{x^3 + x + 1}{x\,(x^2+1)}$ DO NOT integrate the result.

(16) Evaluate each of the following integrals

a. (5 points) $\int \sin(x+2)^3 \, dx$

b. (5 points) $\int \sin(3\,x)\cos(7\,x) \, dx$

(17) 10 pts Evaluate each of the following

a. (5 points) $\int \frac{1}{1 - \sin(\pi\, x)} \, dx$

b. (5 points) $\int_{-\frac{1}{3}}^{\frac{1}{3}} \frac{1}{\sqrt{1 - 9\,x^2}} \, dx$

(18) Give an example of a series $\sum_{i=1}^{\infty} a_i$ such that $\lim_{i \to \infty} a_i = 0$ and yet $\sum_{i=1}^{\infty} a_i$ does not converge). You must explain why it does not converge

19. **SHOW YOUR WORK**. A car travels counterclockwise around a circular track at a constant speed, circling the track six (6) times per hour. The track has a radius of 2 miles. If the track is assumed to be the graph of $x^2 + y^2 = 4$ and at time t=0 the car is at (0,2), then the position of the car at time t hours is given parametrically by

(4 points) x(t) = ____

(4 points) y(t) = ____

(2 points) The position of the car at time $t = \frac{\pi}{4}$ is (______________ , _________________)

20.

6. Calculate the following limits

c. $\lim_{x \to 0} (1 - 3x)^{\left(\frac{3}{x}\right)}$ = __________________

d. $\lim_{n \to \infty} \sqrt{n^2 + 7} - \sqrt{n^2}$ = ________________

e. $\lim_{x \to \infty} \frac{1}{x \arctan(x)}$ = ________________

f. $\lim_{x \to \infty} \frac{7x^2 - 4x + 10}{(2x - 1)(1 - 3x)}$ = _______________