An Embedded Professional Development Model for Secondary Mathematics Teachers with an Alternative Approach to Dual Credit

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Basic Program Premise

Secondary math teachers can successfully serve as teaching assistants for college algebra, precalculus, elementary calculus.

- They substantially have math degrees and could plausibly be math graduate students
- The 18-hour SACS rule does not apply if *:
 - They have no control over the syllabus
 - They have no control over the text
 - They do not assign the homework
 - They do not make out the tests
 - They are under general faculty supervision and meet regularly (weekly) with supervisors.
- They are trained in instructional methods appropriate for sr. high school/early college students

* Checked with UK SACS compliance officer

Program Features

- IHE develop courses and materials
 - Courses are offered on campus
 - Students get free texts and supplemental materials
- IHE faculty mentor High School teachers to offer the courses to advanced HS students
- High School teachers provide instruction
- IHE rigidly controls text, syllabus, homework, exams
- Successful students earn college credit for college course

- High schools free to offer high school credit

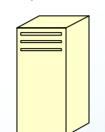
Program made possible through technology

Basic Model Features

Weekly Teacher Seminar/Graduate Course



Central site for services



Students do common online homework and take common exams with on-campus students

Teachers employ appropriate Instructional model: formal class, personal instruction, mentoring, tutoring, etc. for their students.



Students complete program:

- With credit in a principal "failout" course
- With a college transcript and an established relationship with the IHE
- Not having used any state scholarship (KEES) money
- Prepared for further credit-bearing math courses
- Experienced with distance learning mathematics instruction

Economics

- Third party (NSF, Foundation, State) pays program operations:
 - Faculty: 1 FTE to develop, maintain a course
 - technology: approx \$100,000/yr for licenses, staff, operations
 - materials approx \$25 per student
 - development
 - distribution,
 - exam scoring, approx \$20/student
 - teacher stipends, approx \$1500/yr
 - Overhead 25%
- IHE grants credit (no tuition/no fees) to successful students - \$500-\$900/student
- Instruction is off-campus
- IHE is credited with instruction
 - Undergraduate. Graduate (fpr teachers)

Nominal costs/yr for a model program

- 50 teachers = \$100,000/yr
- 1000 students = \$100,000/yr
- Evaluation/assessment = \$100,000
- Technology = \$100,000
- Faculty/grad students = \$400,000
- Overhead = \$200,000
- \$4000/teacher/year, \$200/student/yr

Evaluation

- Content and technology evaluation is provided through direct feedback from teachers to developers.
 - Continuous improvement
- Program evaluation is provided by student homework and examination scores relative to those of the (on-campus) comparison groups

 Maintenance of academic standards

Program as Technology Transfer

- Program is treated as systematic transfer of advanced technology from developers to professionals
- Employs year-long (weekly) seminar format:
 - moderated by expert user(s)
 - participants include recipients, developers and users
 - incremental transfer of content, philosophy, and implementation tools
 - Incremental implementation with discussion of evaluation issues
 - evaluation and improvement of the technology a major goal
- Program (objectively) assessed through measures of implementation success
- Program develops cadre of instructors familiar with IHE courses
- Provides formal process of curriculum alignment

'Academic' Technology Transfer

- Technology is a complete, "new" course of instruction
 - Has been successfully taught in one context
 - Objective is to transfer to participants the capacity to successfully offer (appropriate versions of) the course in their context
 - Free text, free online homework
- Academic year seminar
 - Includes participants and developers.
 - Moderated by expert user in "production" environment
 - Begins with overview and "walkthrough" followed by weekly discussion of instructional objectives, methods, and progress
 - Can be offered for graduate credit

Technology

- (Centra) Asynchronous instruction (commercial system)
- Webclass (open source, developed with AMSP support at UK)
 - WHS (instructional support)
 - Online Homework
 - formatted math
 - Student/teacher interaction
 - Testing
 - Course
 - » Early (KEMTP)
 - » Placement
 - Materials Development tools
 - Posting System
 - Integrated web server
 - Problem Writing systems
 - Diagram development tools
 - Support for other disciplines
 - languages

An Example:

Access to Algebra

- College Algebra for high school students as corollary to professional development for teachers
- Alternative approach to dual credit
- Differential pay model for secondary math teachers
- Partnership among multiple independent organizations, programs
- Run by Lee Alan Roher as part of her doctoral research program
- Primarily supported by the AMSP

AY (semester) Format

- Teachers select, invite average of 4 students for algebra program
- Students take UK Ma109 (college algebra)
 - Counselors must approve on behalf of school
 - Parents must approve
 - Same syllabus, (online) homework, schedule, tests as comparison groups of college students
 - Local Teacher is primary resource
 - Course parallel to defined on-campus comparison group
- Tests administered by teacher and <u>sent** to UK for common</u> grading
 - Same tests as comparison group with same graders

- Course is UK Ma109 (College Algebra)
- Lee Roher is teacher of record.
 - Participating teachers are formally teaching assistants for recitation sections

- Cleared by UK SACS officer

- Students are not registered until it is clear that they can succeed. Are granted the option of accepting credit with earned grade or not. <u>All</u> registration fees and tuition are waived by the university
- Teachers meet online weekly to discuss course, upcoming material, student progress, etc.

Free* Text Written by UK Math Professor

https://www.msc.uky.edu/sohum/ma110/text/ma110_fa07.pdf

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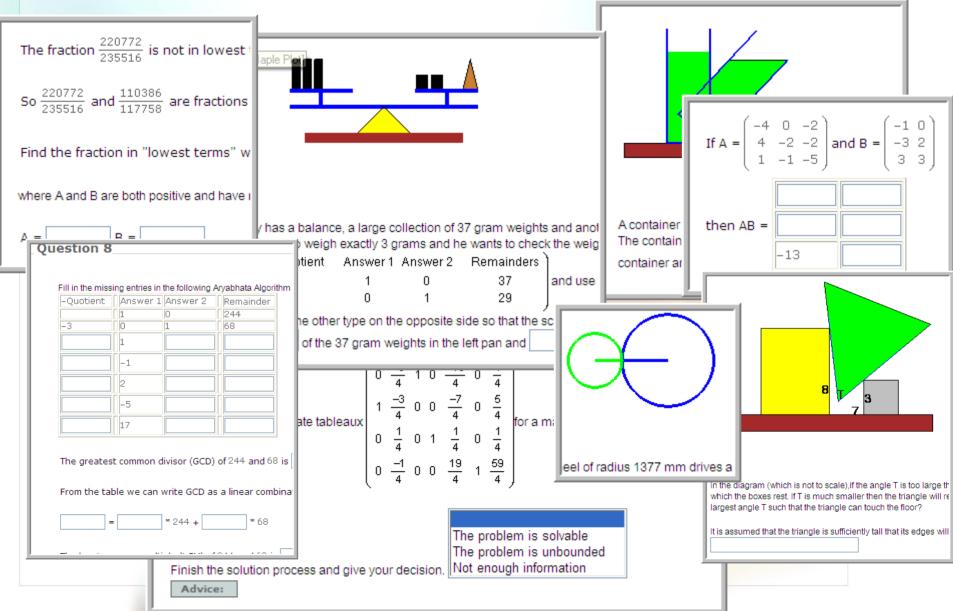
11.2 Limitations of the Newton's Method

Free Web Homework with Video Solution:

Videos Prepared by UK Faculty and Pre-service math teachers through support from AMSP

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Employs Open Source Homework System Developed with Support from AMSP



Formatted Math Feedback allows students to direct											
specific questions about homework problems to teacher and/or instructional assistants.											
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$g^{2}-11 = 2\sqrt{30} \text{ for convenience } x = g^{2}-11 \text{ then } x^{2} = 4.30, x^{2}-4.30 \text{ since this is irreducible then } x^{4}-22x^{2}+1 \text{ is irreducible.}$ $# # WHAT THIS SHOWS IS THAT THERE IS NO LINEAU FACTOR. YOU STILL HAVE TO SHOW THAT THERE IS NO QUADRATIC FACTOR.$ $\{although after trying to work with it I'm still confused how this proves irreducibility and believe I am missing the tail end of this concept} \\ # # THE IDEA IS THAT IF \sqrt{4} \cdot 30 = x IS AN INTEGER THEN x^{2} - 4 \cdot 30 IS REDUCIBLE. SINCE IT IS IRREDUCIBLE THERE IS NO SUCH x SO x - \sqrt{s} - \sqrt{t} CANNOT BE A LINEAR FACTOR OF x^{4} - 22x^{2} + 1. < \boxed{[Style] \ (Font] \ (Size) \ (MA AM)} \\ B I U S x^{2} x_{2} \equiv a \equiv a = -a \equiv a = a $	e CT(Pizzuti, Michael	E	14:01-6	Answered: $x^4 - 2(s+t)x^2 + (s-t)^2 = x^4 - 2(11)x^2 + (1)^2 = x^4 - 22x^2 + 1$ To prove this is irreducible we use ($x - \sqrt{s} - \sqrt{t}$ & wnbsp;) ($x - \sqrt{s} + \sqrt{t}$) ($x + \sqrt{s} - \sqrt{t}$ & wnbsp;) ($x + \sqrt{s} + \sqrt{t}$ & wnbsp;) = 0 so $g = \sqrt{s} + \sqrt{t}$, $g = \sqrt{6} + \frac{1}{2}$ then $g^2 = 5 + 6 + 2\sqrt{30}$ $g^2 - 11 = 2\sqrt{30}$ for convenience $x = g^2 - 11$ then $x^2 = 4.30$ making $x = \sqrt{4.30}$ as an integer and then $x^2 - 4.30$ is irreducible. Because it is irreducible then $x - \sqrt{s} - \sqrt{t}$ cannot be a linear factor. *The quadratic factors can be $1, x^2 - (\sqrt{6})(\sqrt{5}) - 11$ or $2, x^2 - 2x\sqrt{6} + 11$ or $3, x^2 - 2x\sqrt{5} - 1$ One of the above can only be a quadratic factor because since we are working in Z the must be only whole integer. Number $2 \sqrt{6}$ cannot be a whole integer. Number $2 \sqrt{6}$ cannot be a whole integer. Number $2 \sqrt{6}$ cannot be a whole integer. Since there can be no quadratic factor in Z_x since both 2 and 3 are pri- numbers their square root would result in an irrational number in Z_p not a whole integer which is what is needed. The same principle applies to number 3, the $\sqrt{5}$ cannot be will integers because five is a prime number. Since there can be no quadratic factor in $Z[x]$ and there is no linear factor the polynom is irreducible in $Z[x]$. The polynomial factors in mod 11 due to $x^4 - 22x^2 + 1 = x^4 + 18$ mbsp; using $t = 4^2$ The factors are $x^4 + 1 = (x^2 + 3x + 10)(x^2 - 3x - 1)$ Although the polynomial will factor in mod 13 and $x^4 + 9x^2 + 1 = (x^2 + 7)(x^2 + 2)$. Comment: Manual Grading (12/7 1:37) (Grading Requested)						
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(although after trying to work with it I'm still confused how this proves irreducibility and believe I am missing the tail end of this concept) ##THE IDEA IS THAT IF `sqrt(4*30) = x ` IS AN INTEGER THEN `x^2 - 4*30` IS REDUCIBLE. SINCE IT IS IRREDUCIBLE THERE IS NO SUCH x SO $ x - \sqrt{s} - \sqrt{t}$ CANNOT BE A LINEAR FACTOR OF `x^4-22x^2+1`. <											

System supports secure, formatted open response

math exams with intuitive, 'calculator syntax' expressions

Question 1

Use Eisenstein's criterion to prove that

a. Problem statement

f(x) =

 $1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{16} + x^{16} + x^{17} + x^{18}$ is irreducible in Q[x].

One way to do this is to note that $f(x) = \frac{x^{19}-1}{x-1}$. This is just the geometric series. Replace x by x+1 expand by the binomial theorem and simplify. Argue that Eisenstein's criterion applies to the resulting polynomial. Note there is no need to actually expand the expression, one is only interested in the form that the result takes.

 $(x^{19} - 1)$, $(x - 1) = 1 + x + ... + x^{18}$

Eisenstein's Criterion

c. Formatted student response Replace x by x+1 $\frac{(x+1)^{19}-1}{x-1+1} = \frac{x^{19}+19x^{18}+\binom{19}{2}x^{17}+...+19x+1-1}{x}$

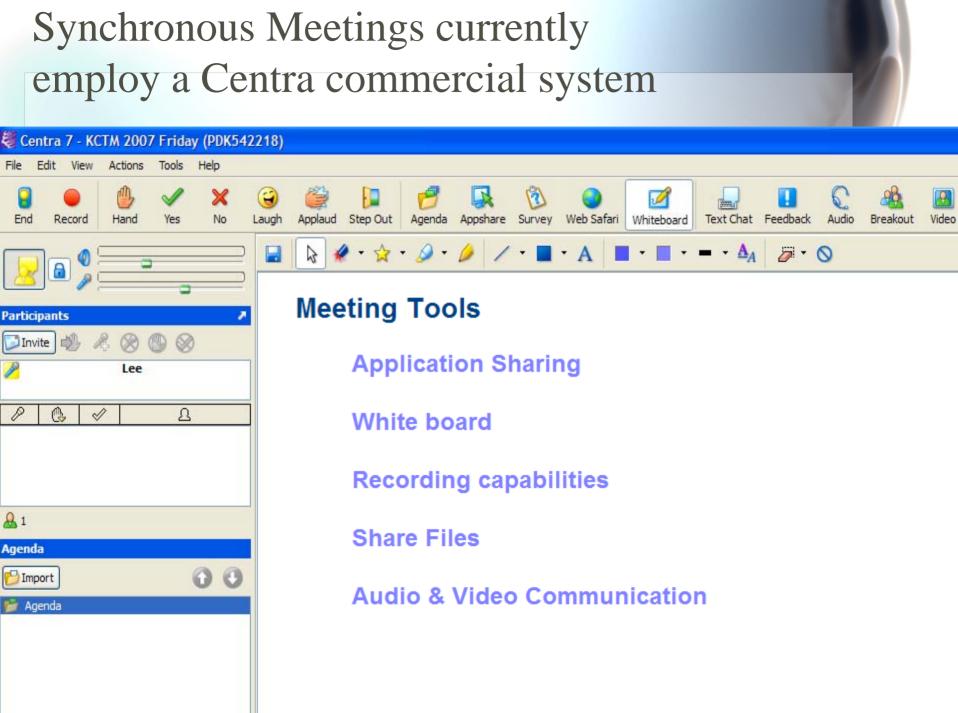
= $x^{18} + 19x^{17} + {19 \choose 2}x^{16} + ... + 19$ which is irreducible by Eisenstein

d. Teacher response in mathml editor This is ok as far as it goes but you need to explain how you are applying Eisenstein's Criterion

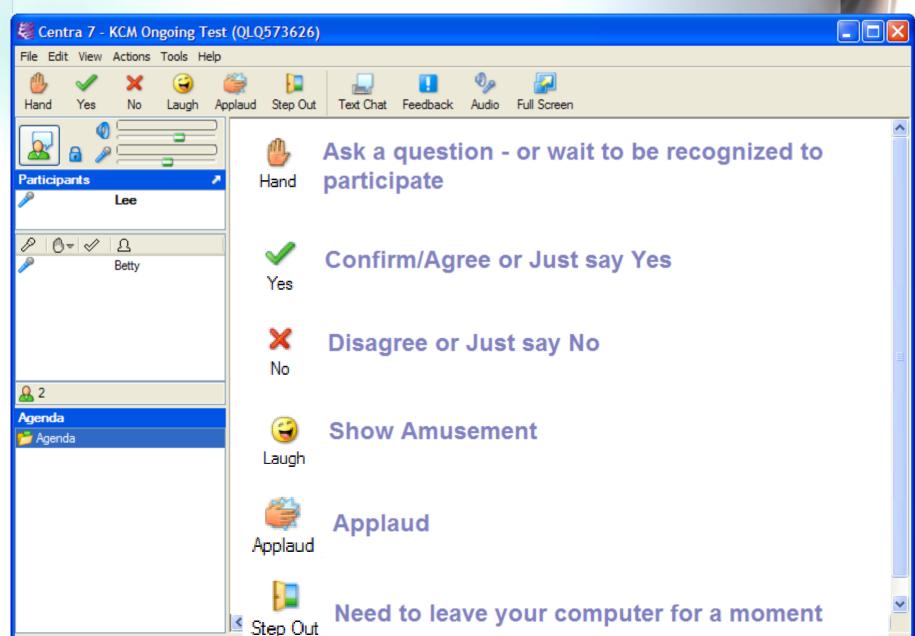
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`(x^	19 -	·1).((x-1)) `	= `	1+x	+.	. +	x^18	3`														
b. Student response in mathml editor Replace x by x+1 $((x+1)^{19} - 1)/(x-1+1) = (x^{19} + 19x^{18} + ((19),(2))x^{17} ++ 19x + 1-1)/x$																								
= $x^{18} + 19x^{17} + ((19),(2))x^{16} + + 19$ which is irreducible by Eisenstein																								
This is ok as far as it goes but you need to explain how you are applying																								

d. Teacher response in mathml editor

This provides the capacity to economically and efficiently administer, "collect" and return open response math exams with fully formatted responses

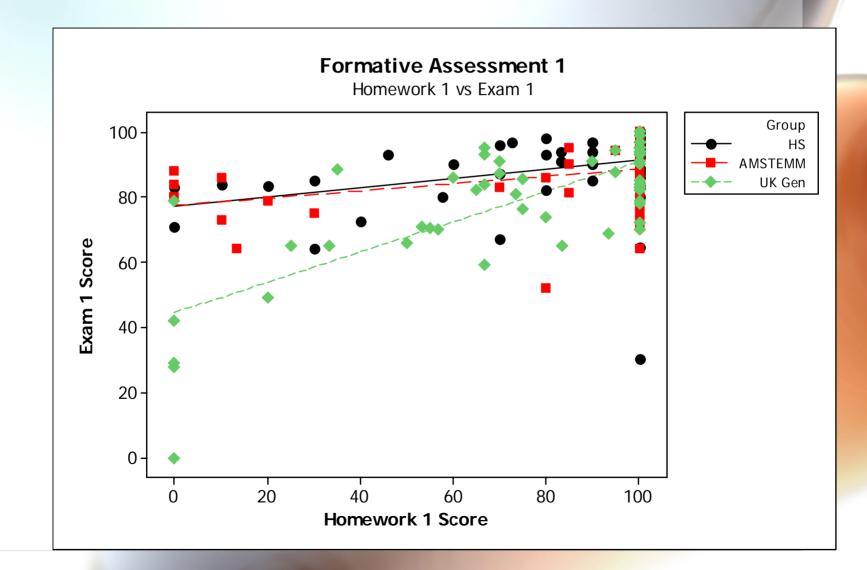


Meeting tools

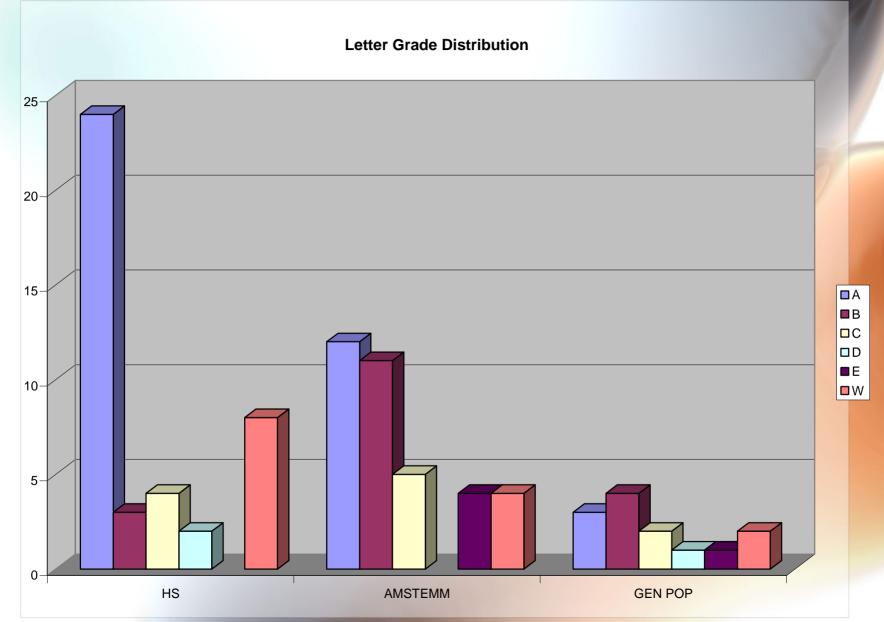


Outcomes

Representative Formative Evaluation Data: Student scores on exam 1 vs online homework participation Spring 2007



Fall 2006 Results: Letter Grade Distribution



Student Outcomes: Fall 2006

	HS (41)	Comp Gp (45)	Random (13)	Gen. Course F05 (1686)
Α	60 %	33 %	23 %	29 %
B	8 %	27 %	31 %	19 %
С	10 %	18 %	15 %	13 %
D	5 %	0 %	8 %	6 %
Ε	0 %	16 %	8 %	9 %
W	18 %	7 %	15 %	19 %

29 of 41 (70%) of the HS students received a college transcript with credit for college algebra.
(two C's declined the credit)

Student Outcomes: Spring 2007

Spring 2007: Final Grades for High School (HS), Comparison group CMP), General students (GEN), and General Course* (CRS)

(===-,,		<u> </u>		
	HS	CMP	GEN	CRS*
	(N= 24)	(N= 10)	(N= 50)	(N= 663)
Α	50%	60%	32%	17%
В	33%	10%	16%	21%
С	4%	20%	16%	18%
D	0%	0%	4%	14%
F	4%	10%	12%	17%
W	8%	0%	18%	14%

Percentages are rounded to the nearest integer so the columns may not total 100%.

*The general course uses a conventional text and both large lecture and individual section formats. The "GEN" group was self-enrolled from the same pool as the general course. Assessment of the effectiveness of the PD program for participating teachers (Spring 2007 data)

Teachers	Total	HS Students	HS Students Students receiving College Algebra credit	Success Rate
Systematic Participation	11	21	19	90%
Non-systematic Participation	3	9	3	33%*

Program Features:

- Articulation with other NSF programs (AMSP and AMSTEMM)
- Application of advanced distance learning and instructional support technology
- Integrated STEM and Education research programs
 - (open source) materials and technology developed for the program by research faculty
 - Program evaluation is leading to improvements in technology
- Operates though traditional university and school instructional structures and faculty assigned roles at costs that are modest and easily understood in these contexts.
- Integrated evaluation and assessment
- Program technology, materials, methods can be transferred to others employing the same basic tools

Some Images From the Project

Access to Algebra Startup (part B)



CATSbusters: Summer 2005 Graduate Course







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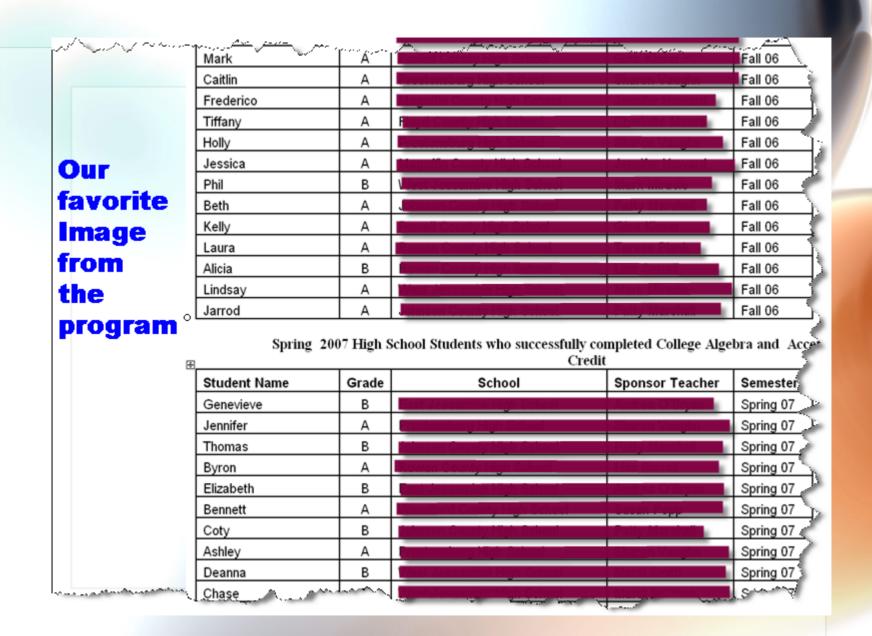
W. Anderson

Agtern Ditty //www.mac.uky.adu/k.en/ma801.moter./logic Algebra for Teachers 4 Ø

Logic Class Notes

Ele Edit View Egypties Tools Help

These notes are an indexial in Chapter D, section 1 of the <u>on-line Incident</u>, The corresponding WHS exercises are 02, and 024. Clask on the items below to doplayfinde the corresponding note. - Notes Pape 1 - Notes Pape 2 - Notes Pape 3 - Notes Pape 4 - Of CRUMMER, PERFORMENCE Drivena Privation and a) Pisonstady (74 (5x3)) = 2.2 Ordan Spectra answer Chinad Break answer 4) West Sweet Style Dave Anna Sanly 4) West <u>Visonary Misons</u> Misonald UBARIES Minnaft VBANIAS - 2" = 5(2)" if u-rofi (- 2") otherwise - 24 - 516 CPERAME AssociATIVITY. LEFT ASSUC. (A + B)+C) Right Assoc. (1(1A)) $a_{+}(b=2)$ Non house $\Rightarrow 2^{2} + 52^{2}$ ((22)² "ADD PARENTINGERS FOR CLARING



With the exception of the Centra conferencing system, the intellectual property employed in this program was developed in open source by the University of Kentucky Mathematical Sciences through grants and contracts funded by :

- The National Science Foundation
 - Primarily through the Appalachian Mathematics and Science Partnership (AMSP)
- The US Department of Education
- The Ky Department of Education
- The Ky Council on Postsecondary Education
- The University of Kentucky

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